

Exercise 17

Find the limit.

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x)$$

Solution

Evaluate this limit by writing the difference as a quotient.

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x) &= \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x) \times \frac{\sqrt{x^2 + 4x + 1} + x}{\sqrt{x^2 + 4x + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 4x + 1} - x)(\sqrt{x^2 + 4x + 1} + x)}{\sqrt{x^2 + 4x + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + 4x + 1) - x^2}{\sqrt{x^2 + 4x + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{4x + 1}{\sqrt{x^2 + 4x + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{4x + 1}{\sqrt{x^2 \left(1 + \frac{4}{x} + \frac{1}{x^2}\right)} + x} \\ &= \lim_{x \rightarrow \infty} \frac{4x + 1}{x \sqrt{1 + \frac{4}{x} + \frac{1}{x^2}} + x} \\ &= \lim_{x \rightarrow \infty} \frac{4 + \frac{1}{x}}{\sqrt{1 + \frac{4}{x} + \frac{1}{x^2}} + 1} \\ &= \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} \\ &= 2 \end{aligned}$$